TESTING THE EFFECTIVENESS OF THREE-STEP APPROACHES FOR AUXILIARY VARIABLES IN LATENT CLASS AND LATENT PROFILE ANALYSIS

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Latent class analysis (LCA) and latent profile analysis (LPA) can be used to find unobserved groups, known as classes, that are different based on observed characteristics, estimate probabilities of individuals belonging to each class, identify characteristics that predict class membership, and estimate differences in distal outcomes between classes (Bolck, Croon, and Hagenaars, 2004). Members of a particular class have similar profiles of scores; however, these members have different profiles compared to members belonging to other classes. LCA and LPA can be exploratory or confirmatory (Finch & Bronk, 2011). This study focuses on the most widely used type, exploratory LCA and LPA.

LCA and LPA are often used in educational research, sociology, psychology, and survey inquiry by researchers such as Keel et al. (2004), Klonsky and Olino (2008), Berge et al. (2010), and Sperrin et al. (2014). For example, Denson and Ing's (2014) study used LCA to classify entering college freshmen based on their pluralistic orientation at the start of college. Denson and Ing (2014) examined whether the latent classes were related to students' demographic and background characteristics, and provided suggestions on how LCA can aid college administrators in their program planning and targeting of interventions. Chung, Flaherty, & Schafer (2006) investigated how class membership relates to demographic and lifestyle factors, political beliefs, and religiosity over time in regards to marijuana usage.

While the inclusion of auxiliary variables in LCA models has become more common in applied research, there have been few studies comparing methods to include auxiliary variables in LCA. Furthermore, including auxiliary variables in LPA models has not been addressed in previous research. Issues concerning auxiliary variables in LCA are dependent upon the type of auxiliary variable associated with the model:

1) *Predictor auxiliary variable* (latent class regression analysis), the latent categorical variable is predicted by the observed variable;

2) *Distal outcome*, the observed variable is predicted by the latent categorical variable;

Even though applied researchers have compelling reasons to examine the relationships between covariates and latent classes, problems related to including both types of auxiliary variables in mixture modeling include unwanted influence in class membership, increased computation time, incorrect estimates, and incorrect standard errors (Asparouhov & Muthén, 2014). With this in mind, each issue is attributed to the traditional method of including auxiliary variables in LCA models, the “single-step” approach.

Several 3-step approaches were developed in order to autonomously assess the association between the latent categorical variable and the distal or predictor auxiliary variable (Asparouhov & Muthén, 2013). One such method is the pseudo class (PC) method designed for predictor auxiliary variables and distal outcomes. This method entails estimating the LCA model, then the latent class variable is assigned from the posterior distribution obtained from the first step, and the assigned class variables are evaluated with the auxiliary variable (Asparouhov & Muthén, 2013).

The second approach discussed in this study is Vermunt’s method (2010) for predictor auxiliary variables and distal outcomes. Vermunt’s method estimates the LCA model and obtains a most likely class variable from the posterior distribution in the estimated LCA. From the previous step, the most probable class variable indicates the latent class variable with uncertainty rates preceded at the probabilities (Asparouhov & Muthén, 2013). Vermunt's approach was developed based on ideas modeled by Bolck et al. (2004).

The final approach discussed in this study was developed by Lanza et al. (2013) solely for distal outcome auxiliary variables. In this method, the probability distribution that gives the likelihood of the latent class variable and the distal variable is regressed by the latent class variable as well as conditioned by the distal outcome variable, jointly with the distal variable's marginal dispersal (Asparouhov & Muthén, 2014).

Little research has determined the best usage of the PC method, Vermunt's method, and Lanza's method for using auxiliary variables in LCA. This is an issue because each of the methods has their own assumptions and there is little information in the literature about required sample sizes for these methods, how they compare with respect to power, and whether they provide unbiased estimates across a variety of conditions. In addition, the lack of literature may discourage future studies from including LCA with covariates with real data. The purpose of this study is to compare the PC, Vermunt's, and Lanza's method in order to define the conditions under which each approach is appropriate. This study will expand the existing simulation studies by manipulating sample size, entropy, and the strength of the relationship between the auxiliary variable and latent class.

Latent Class and Latent Profile Analyses

The difference between latent profile analysis (LPA) and latent class analysis (LCA) is that outcome variables are continuous in LPA, and are categorical in LCA.

The LCA model for two indicators can be expressed as:

$$\pi_{ijt}^{ABX} = \pi_{it}^{\bar{A}X} \ \pi_{jt}^{\bar{B}X} \ \pi_t^X \tag{2-1}$$

where π_{ijt}^{ABX} is the likelihood that an item will be located in the i,j,t cell, $\pi_{it}^{\bar{A}X}$ is the conditional likelihood that an indicator in class t of the latent variable (X) will be located at level i of variable A, $\pi_{jt}^{\bar{B}X}$ is the conditional probability of being at level j of variable B, and π_t^X is the chance of a randomly selected indicator being at level t of the latent variable X (Henry, 1983). A LCA with *m* indicators is:

$$\pi_{ij\ldots mt}^{AB\ldots EX} = \pi_{it}^{\bar{A}X} \ x \ \pi_{jt}^{\bar{B}X} \ x \ldots x \ \pi_{mt}^{\bar{E}X} \ x \ \pi_t^X \tag{2-2}$$

A LPA model for observed variable A can be expressed as:

$$\sigma_A^2 = \sum_{t=1}^{T} \pi_t (\mu_{At} - \mu_A)^2 + \sum_{t=1}^{T} \pi_t \sigma_{At}^2 \tag{2-3}$$

where μ_{At} and σ_{At}^2 denote (t) class-specific means and variances for variable A, and π_t show the proportion of N participants that belong to class t.

Auxiliary Variables

The latent categorical variable can be further analyzed by investigating the association between that variable and other auxiliary indicator variables (Asparouhov & Muthén, 2014). In LCA and LPA, the auxiliary variable can be modeled as a predictor of the latent categorical variable, or the latent categorical variable can be used as the predictor of the auxiliary variable. In the first case, the variable is a *predictor auxiliary variable*, which in the second case it is a *distal outcome*.

In studies where the covariate is categorical, a chi-square test can be used to check whether frequencies of the auxiliary variable are the same across each of the classes (Dakdjd, 2014). The chi-square test is:

$$Q_P = \sum_{j=1}^{K} \frac{\left(f_j - e_j\right)^2}{e_j} \tag{2-11}$$

where f_j represents the frequency of the auxiliary variable in class *j* (or the total observations in class j) for j = 1, 2, . . ., *K* (Clark & Muthén, 2009). As a bi-product of the null hypothesis specifying like proportions of the entire sample size for each j, the likely frequency for each j equals the full sample size ,n, divided by the number of classes, or:

$$e_j = n/K \text{ for } j = 1, 2, \ldots, K \tag{2-12}$$

LCA modeling with an auxiliary variable, x, where the likelihood that individual i, falls in class k of the latent class variable X is expressed through multinomial logistic regression as

$$P(X_i = k|x_i) = \frac{e^{\alpha_k + \gamma_k x_i}}{\sum_{s=1}^{K} e^{\alpha_s + \gamma_s + x_i}}, \tag{2-13}$$

where $\alpha_K = 0, \gamma_K = 0$ so that $\alpha_K + \gamma_K x_i$ = 1, which implies that the log odds of comparing class k to the last class K is

$$\log\left[\frac{P(X_i = k|x_i)}{P(X_i = K|x_i)}\right] = \alpha_k + \gamma_k x_i . \tag{2-14}$$

Evaluation of Model Fit

When evaluating overall fit of a LCA or LPA model, a grouping of relative entropy, information criteria, and likelihood ratio tests have been used. The Akaike information criterion (AIC) and Bayesian information criterion (BIC) adjust differently for the number of parameters (p) and sample sizes (N) (Lubke & Muthén, 2005; Lubke & Neale, 2006.) Models resulting in lower AIC and BIC indicate better fit.

Relative entropy, labeled “Entropy” in Mplus, has a range that starts at zero and increases to one. Entropy can be computed as

$$E = 1 + \frac{1}{Nlog(t)} (\sum_{i=1}^{N} \sum_{t=1}^{T} P(C = t|U_i) \log (P(C = t|U_i))) \quad (2\text{-}17)$$

where C is the latent variable, K is the number of classes, N is the sample size and U_i is the vector of all latent class indicator variables and the probabilities $P(C = t|U_i)$ are computed from the estimated model (Asparouhov and Muthén, 2014b). Unlike AIC and BIC, higher relative entropy indicates better model fit.

Likelihood Ratio Tests

Although LCA and LPA models with different numbers of classes are nested models, the chi-square difference test in the form of the likelihood ratio test (LRT) is not appropriate in this kind of analysis (Nylund et al., 2007). When applying this method, the p value obtained would not be accurate. Therefore, when figuring the difference in likelihoods of a $t - 1$ class and a t class model, the difference is not Chi Square distributed and standard difference testing is not appropriate (Nylund et al., 2007). In LCA and LPA, the Lo-Mendell-Rubin likelihood ratio test and Bootstrap likelihood ratio test can be performed instead of the traditional likelihood ratio test. Both of the Vuong-Lo-Mendell-Rubin and Bootstrap likelihood ratio tests reject or fail to reject the null hypothesis (H_0), the estimated model. If either of these likelihood ratio tests results in a p value less than .05, the test favors the estimated model over the model with one fewer class (H_1).

Conducting Analysis with Auxiliary Variables

According to Asparouhov and Muthén (2014), the standard method of dealing with auxiliary variables in LCA is by combing the LCA model and the auxiliary model (distal or predictor auxiliary variable) into a joint model. As a result, this method can be predictable with the maximum likelihood estimator and is called the “one-step” and/or “single-step” approach. Using the one-step approach in LCA is faulty, because the subordinate model affects the latent class construction. Furthermore, the latent categorical variable may lose its meaning as the latent variable measured by the indicator variables (Asparouhov and Muthén, 2014).

Since the discovery of these flaws within the one-step method, several methods have been established in order to autonomously assess the relationship between the latent class variable and the predictor or distal auxiliary variables (Asparouhov and Muthén, 2014). The methods that we will discuss are the pseudo class (PC) method (1987), Vermunt’s method (2010), and the Lanza (2013) et al. method.

The Pseudo Class Method

The PC method makes draws by taking random samples for the multinomial distribution, which allows individuals opportunity to join different classes. Draws from the PC method allow class specific means, variances, mean equality tests and regression to be determined (Clark and Muthén, 2009).

Asparouhov and Muthén (2014) describe the steps to the PC method as follows:

1. Estimate the LCA
2. Multiply impute the latent class variable from the posterior distribution obtained from the LCA model estimation
3. Use the multiple imputation technique developed in Rubin (1987) to analyze the imputed class variable with the covariate

For more detailed information on the third step and development of the PC method, review Scafer (1999). According to Clark and Muthén (2009), the PC method underestimates the regression effects of the classes on covariates; however, the method performed well when entropy was above .80.

Vermunt's Method

In 2010, Vermunt modified what he refers to as the "BCH" method created by Bolck, Croon, and Hagenaars (2004). The BCH method is flawed, because it requires the covariate to be categorical, matrix multiplications are required in the arrangement of data stage and must be repeated after the addition of new covariates, and analyzing the reweighted data using a standard logistic routine yields severely downward biased standard errors; thus, too liberal a significance test for the logistic regression coefficients (Vermunt, 2010).

Vermunt's proposed steps are as follows:

1. Estimate the latent class model using only indicator variables
2. Create the maximum probable class variable using the latent class posterior distribution attained in step one
3. Regress the most likely class on predictor variables taking into account the misclassification of the subsequent step

Standard errors may be slightly underestimated because the classification error probabilities $P(W = s|X = t)$ are treated as known; although they are determined with the estimated parameters of the LCA model without auxiliary variables (Vermont, 2010).

Lanza's Method

The Lanza et al. (2013), method has the following steps:

1) Estimate the LCA model.

2) Estimate a model with auxiliary variables where the distal outcome is used as a latent class predictor inside a multinomial logistic regression in addition to the latent class model

3) To determine the conditional and marginal distributions use the auxiliary model

We will present the third step in more detail. Applying the Baye's theorem below can derive the desired conditional distribution:

$$P(D|C) = \frac{P(D)P(C|Z)}{P(C)} \quad (2\text{-}29)$$

Lanza's method can only be used for distal outcomes. Additionally, Lanza's method cannot have a LCA model that has latent class predictors (Asparouhov and Muthén, 2014).

The only difference between LCA and LPA is the classification of indicators of the latent variable as categorical or continuous. Auxiliary variables can be either categorical or continuous and each 3-step approach handles both types of distal outcome variables. Additionally, each method is initiated in the final step of the analysis and does not assume that indicators are either categorical or continuous. Therefore, even though research on 3-step approaches has been focused on LCA, they can also be used for LPA models.

Research Questions

In this study, the PC method and Vermunt's method, and Lanza's method are compared after manipulation of sample size, the strength of the association between auxiliary variable and latent class, entropy, and type of auxiliary variable. Additionally, we provide a step-by-step guide for running a LPA with estimation of a distal outcome using each of the three-step methods. The research questions addressed by this study are:

- **RQ1**: How do the PC and Vermunt's three-step methods perform with respect to estimating the coefficient of a predictor of class membership and its standard error with varying levels of sample size, class separation, and strength of predictive relationship?
- **RQ2:** Do the PC, Vermunt, and Lanza three-step methods differ with respect to power and type I error to test the estimated relationship between the auxiliary variable and the latent class variable?
- **RQ3:** What are the differences in implementation and results between the PC, Vermunt, and Lanza three-step methods when applied to a dataset with a distal outcome?

Table 2-1: Two- Class LCA Example

	X = 1 (Tolerant)	X = 2 (Intolerant)
P(X = x)	.62	.38
P(Y1 = 1|X = x)	.96	.23
P(Y2 = 1|X = x)	.74	.04
P(Y3 = 1|X = x)	.92	.24

Table 2-2: Auxiliary Variable Table Example

Y1	Y2	Y3	Y4
1	1	1	14
1	1	2	23
1	2	1	55
1	2	2	70
2	1	1	17
2	1	2	29
2	2	1	37
2	2	2	46

METHOD

Population Model for Simulation

Data for two-class models with five binary observed variables were generated using R for this Monte Carlo simulation study. A simulation was used because analytical comparisons between methods are not available. In this study, the described model was chosen, because similar two-class models have frequently been used in applied research with LCA such as Pickels et al (1995), Klonsky and Olino (2008), and Ubersax and Grove (1990). Furthermore, previous methodological studies have been limited in quantity and also in the manipulations of conditions of two-class LCA models. As in Asparouhov and Muthén's (2014) study, each binary observed variable U's distribution was determined by the logit relationship

$$P(U = 1|C) = 1/(1 + Exp(k_c)) \tag{3-1}$$

where C is the latent class variable which takes values 1 or 2 and the threshold value τ_c is the same for all 5 binary indicators. In addition, we set $K_2 = -K_1$ for all five indicators.

Next, an auxiliary variable was generated as a predictor of the latent class variable by using a logistic regression model

$$P(C = c|X) = 1/(1 + Exp(\alpha + \beta X)) \tag{3-2}$$

where the population value of α is held constant at 0.3. Also, β denotes the effect for the relationship between the class membership and the auxiliary variable *X*. The auxiliary variable was normally distributed, had a mean of zero, and a standard deviation of one.

Manipulated Conditions

Manipulating the threshold value directly alters the overall separation of the classes, measured by entropy, by changing the probabilities describing the probable outcomes of a single trial, as a function of the predictor variables. This logistic relationship manipulates $P(C = k_c|U_i)$ within the entropy equation, Equation 2-14, previously discussed. Thus, a larger specified k_c value will result in a larger separation of classes. The opposite is true when the k_c value is smaller. Using the value of $\boldsymbol{k_1}$= 1.50, we obtained an entropy of 0.8 and with value $\boldsymbol{k_2}$ = 0.75, we obtained an entropy of 0.5. Simulations of population, N, with 50,000, 100,000, and 150,000 cases were conducted in order to determine the values of entropy. These two values of entropy were chosen, because previous research has shown that when separation increases to .8, logistic regression has the lower values of mean squared error (MSE) (Clark & Muthén, 2009). This study followed the suggestions of that study by making two distinct levels of separation of classes, medium and high.

Next, an auxiliary variable was generated as a predictor of the latent class variable by using a logistic regression model, Equation 3-2, where the population value of α was held constant at 0.3. Unlike Asparouhov and Muthén (2014), this study simulated relationship between the class membership and the auxiliary variable with population values of β equal to 0 for no effect, .3 for medium effect, and .5 for a large effect. The justification for generating conditions with zero effect was to allow for the evaluation of Type I error rates. The justification for generating conditions with effect of 0.3 and 0.5 was to evaluate change of power as the effect becomes larger. To analyze the data, the PC method and Vermunt's method were used in this simulation study.

A normally distributed distal outcome variable was generated with a regression to establish a class that had a mean of zero and the other class to have a mean equal to β. The population effect size of class membership on the distal outcome variable, β, was set to zero, 0.5 and 0.8. Since the difference between the two types of auxiliary variables is the direction of the covariate in the LCA model, the same justification of variation in effect sizes of the association between the class membership and the distal outcome variable apply.

We defined the sample sizes of the simulated datasets as 100, 500, and 5,000 observations, because Asparouhov and Muthén (2014) suggested that future researchers test datasets of these magnitudes since no other studies have done this before. Furthermore, many applied studies, such as Deerwester et al. (1990), Barns et al. (2013), and Collins and Lanza (2010), using LCA models have similar sample sizes. Manipulating these sample sizes may help applied researchers with similar data. Predicted changes across sample sizes are due to its effects on the correctness of estimations of the effect of latent class membership and the latent class model on the distal outcome (Tan et al., 2015).

The factors manipulated in this simulation were a) type of auxiliary variable with two levels, b) sample size with three levels, c) entropy with two levels, and d) relationship between auxiliary variable and class membership with three levels. Therefore, the study is a $2 \, x \, 2 \, x \, 3 \, x \, 3$ design, resulting in 36 conditions. For each condition manipulated, 1000 datasets were simulated. Of the 36,000 datasets generated with an auxiliary variable that is a predictor of class membership, the researcher analyzed with the PC method and Vermunt method. For the datasets generated with an auxiliary variable that is a distal outcome, the researcher analyzed with the PC method, the Vermunt method, and the Lanza method.

Analysis

LCA models were fit with 7.31 version of MPLUS. Each method was compared based on relative bias of coefficient approximations, relative bias of standard errors estimate, Type I error rates, coverage, and power.

Relative bias of coefficient estimates was used as an outcome in this study, because it is more easily interpretable than bias given that it takes the form of a proportion. . Relative bias (Bandalos, & Leite, 2013),

$$rBias(\hat{\theta}_i) = \sum_{j=1}^{n_r} \left(\frac{\hat{\theta}_{ij} - \theta_i}{\theta_i} \right) / n_r \qquad (3\text{-}3)$$

which denotes θ_i as the population parameter, $\hat{\theta}_{ij}$ as the jth sample estimate of the ith true parameter value, and n_r, is the number of replications within the cell. This studies' criteria for acceptable levels of relative bias of coefficient estimate is an absolute value less than or equal to .05, as proposed by Hoogland and Boomsma (1998).

The relative bias of standard errors estimates were calculated as(Bandalos & Leite, 2013):

$$rBias(\widehat{SE})(\hat{\theta}_i) = \frac{\overline{\widehat{SE}_{\hat{\theta}_\iota}} - \widehat{SE}_{\theta_\iota}}{\widehat{SE}_{\theta_\iota}} \qquad (3\text{-}4)$$

where $\widehat{SE}_{\theta_\iota}$ is an estimate of the population value of the standard error of $\hat{\theta}_i$ and $\overline{\widehat{SE}_{\hat{\theta}_\iota}}$ is the mean of the estimated standard errors of $\hat{\theta}_i$ across NR replications. . The empirical standard error $\widehat{SE}_{\theta_\iota}$ is determined by taking the standard deviation of parameter estimates across the iterations. This study's criterion for acceptable levels of bias is absolute relative bias less than or equal to .1, which is proposed by Hoogland and Boomsma (1998).

Similarly to Asparouhov and Muthén (2014), we used coverage, the percentage of iterations where the population value is contained within the confidence interval of the parameter estimated, to evaluate the adequacy of confidence intervals. Coverage is affected by both parameter estimates and standard errors. Collins, Schafer, and Kam (2001) argued that coverage below 90% is problematic.

For conditions with zero effect, we evaluated whether Type I error rate is close to the .05 alpha level, and whether any method inflates type I error rates.

For conditions with non-zero effect, we calculated the power of each 3-step method to detect the effect of the predictor of class membership or the effect of class membership on the distal outcome.

Due to the extent of this simulation design, visual examination of effects of manipulated conditions is difficult. Therefore, we used a mixed-design analysis of variance (ANOVA) and generalized eta squared (G η^2) (Olejnik & Algina, 2003) measure of effect size to sort manipulated conditions with respect to the magnitude of their effect on relative bias of coefficient estimates, relative standard error bias, coverage, type I error, and power. In these mixed-design ANOVAs, the between-dataset factors were entropy, effect, and sample size. The within-dataset factor was type of method, with three levels: the PC method, Vermunt's method, and Lanza's method. The dependent variables were $\left(\frac{\hat{\theta}_{ij}-\theta_i}{\theta_i}\right)$, the relative deviation of the coefficient estimate from the population parameter, and $\frac{SE_{\hat{\theta}_i} - \widehat{SE_{\theta_i}}}{\widehat{SE_{\theta_i}}}$. , the relative deviation of the estimated standard error from the empirical standard error, coverage, Type I error and power. Additionally, the mixed-design ANOVA models included all interactions between the manipulated factors. Substantial effects were determined when G η^2 was above 0.001.

RESULTS

Latent Class Predictor as an Auxiliary Variable

. Tables 3-1 through 3-5 show the effects of the manipulated conditions on each parameter estimate. Tables 3-6 and 3-7 contain the results of the simulation study with an auxiliary variable using Vermunt's 3-step method and the PC method. of provided parameter estimates refers to absolute values.

Relative bias of coefficient estimates. There was a significant main effect of method G η^2 = .148 on the relative bias of coefficient estimates. This tells us that if we ignore the entropy, sample size, and effect, the PC method and Vermunt's method were still different than each other. Both entropy and effect, relationship between the predictor auxiliary variable and class membership, had an effect on relative bias of coefficients estimates as well G η^2 = .072 and G η^2 = .005. Additionally, sample size had an effect of G η^2 = .003. The fact that entropy interacted significantly with the type of method implemented tells us that the methods respond differently to the adverts of entropy at .5 and .8. Additionally, there was an effect of the interaction between sample size and effect. Relative bias of coefficient estimates for the PC method ranged from .03 to .578 and had a stronger level of bias compared to Vermunt's method. Bias for Vermunt's method ranged from .001 to .064.Conditions with an entropy value of .8 had more acceptable levels of relative bias of coefficient estimates than the entropy value of .5 for both methods. As effect increased from .3 to .5, bias increased in all conditions for the PC method and most conditions for Vermunt's method. Relative bias of coefficient estimates increased with sample size for the PC method except when entropy was .8, sample size was 5000, and effect was .5.

Relative bias of standard error estimates. There was a large main effect of effect, methods, entropy, and sample size on relative bias of standard errors G η^2 = .646, .578, .379, and .005. Additionally, there was an effect of the interaction between entropy and method G η^2 = .531, effect and method G η^2 = .053, sample size and method G η^2 = .015, entropy and effect G η^2 = .007, entropy and sample size G η^2 = .004, and effect and sample size G η^2 = .003. The interaction between entropy, effect, and method had an effect on relative bias of standard error estimates. Furthermore, the interaction between entropy, sample size, and methods had an equal effect. Relative bias of standard error estimates increased with effect for the PC method, except when effect was .5, sample size was 500 and 5000. Relative bias of standard error estimates for the PC method ranged from .162 to 2.03. Bias for Vermunt's method ranged between .009 and .71. The PC method showed decrease in relative bias of standard error estimates when entropy was increased from .5 to .8. Bias decreased in most cases when entropy was increased for Vermunt's method. The interaction between entropy and method shows that Vermunt's method had weaker levels of relative bias of standard error estimates between both levels of entropy in comparison to the PC method. Additional interactions show that the PC method had stronger levels of bias between the three levels of effect sizes compared to Vermunt's method, increases in entropy and effect decrease bias, increases in studied samples sizes and effect averaged decreases in bias, increased sample size and effect decreased bias as well.

Coverage. There was a main effect of the relationship between the predictor auxiliary variable and class membership on coverage G η^2 = .103. Coverage was unacceptable for both methods when effect was zero. However, coverage was acceptable at .95 and above when effect was .3 and .5. For all cases where coverage was below 1 at an effect of .3, coverage increased when effect increased to .5. The PC method had coverage that ranged from .778 to .782 and Vermunt's method ranged .76 to .793.

Type I Error. There was a small main effect of methods (G η^2 = .011) implemented and entropy (G η^2 = .002) on type I error rates. The PC method had type I error rates that ranged from 0 to .034 and Vermunt's method ranged .031 to .055. The type I error increased when entropy was increased from .5 to .8 for all cases except when Vermunt's method had an entropy of .8 and sample sizes of 500 and 5000.

Power. Sample size (G η^2 = .389) and entropy (G η^2 = .117) had the largest effects on power. Effect (G η^2 = .042) and methods (G η^2 = .020) had smaller effects on power. Interactions with the largest effects were entropy and sample size (G η^2 = .058) and effect and sample size (G η^2 = .020). Interactions between sample size and method, entropy and method, and entropy, sample size, and method had the smallest effect on power. Increasing sample sizes between 100 and 5000 increased power. However, power was lower than 0.8 for the PC or Vermunt's method when sample size was 100 and 500, except when sample size was 500, effect was .5, and entropy was .8. Power was equal to or above 0.8 when sample size was 5000 for both methods.

Continuous Distal Outcome Auxiliary Variable

Tables 3-8 through 3-12 show the effects of the manipulated conditions on each parameter estimate. Tables 3-13 through 3-16 contain the results of the simulation study with a distal outcome auxiliary variable using the PC method, Vermunt's 3-step method, and Lanza's method. In this simulation study, the three methods were compared based on the same terms as the previous model.

Relative bias of coefficient estimates. The type of method (G η^2 = .279) effected the relative bias of coefficient estimates the most. The interactions between sample size and methods (G η^2 = .004) and entropy and method (G η^2 = .002) had the least effect on the parameter estimate. In Tables 3-13 through 3-16, these values needed to be below .05 in order to meet the standards of bias by Hoogland and Boomsma (1998). The relative bias of coefficient estimates ranged from .001 to .09 for Lanza's method, .705 to .918 for the PC method, and 0 to .239 for Vermunt's method. The PC method had no acceptable levels of relative bias of coefficient estimates, whereas Lanza's method had 8 cases of acceptable bias and Vermunt's method had 7. When sample size increased, relative bias of coefficient estimates decreased for Lanza's and Vermunt's methods and increased when estimated by the PC method. Bias decreased when entropy was increased for Vermunt's and Lanza's methods, except when sample size was 500 and effect was .5 and when sample size was 5000 and effect was .3. Relative bias of coefficient estimates decreased when entropy was increased from .5 to .8 for the PC method, except when sample size is 5000.

Relative bias of standard errors estimates. There was main effects of effect, entropy, method, and sample size on relative bias of standard errors G η^2 = .6, .149, .145, and .101. Additionally, the interactions between effect and method (G η^2 = .754), entropy and method (G η^2 = .605.), sample size and method (G η^2 = .308), and effect, sample size, and method (G η^2 = .178) had large effects. The interactions between effect and sample size (G η^2 = .094), entropy, sample size, and method (G η^2 = .035), entropy, effect, sample size, and method (G η^2 = .024), and entropy, effect, and method (G η^2 = .018) had smaller effect sizes. The smallest effect sizes on relative bias of standard errors were the interactions between entropy and effect, and entropy, effect, and sample size. In Tables 3-13 through 3-16, these values need to be below .01 in order to meet the standards of bias by Hoogland and Boomsma (1998). Relative biases of standard errors estimates were not acceptable for each all conditions studied. However, there was a lower relative bias of standard errors estimates for entropy valued at .8 compared to .5. Relative bias of standard errors estimates ranged from .028 to .522 for Lanza’s method, .068 to .93 for the PC method, and .015 to .313 for Vermunt’s method. Bias increased with increases in sample size. Relative bias of standard errors decreased with increased effect for Lanza’s method, except when entropy was .8, effect was .5, and sample sizes were 100 and 500. Bias increased when effect was increased for the PC method except when entropy was .5, effect was .3, and sample size was 100. Bias fluctuated between increasing and decreasing when effect increased for Vermunt’s method. Vermunt’s and Lanza’s method decreased levels of bias with increases in sample size, except when entropy was .5, effect was 0, and sample size was 5000 and when entropy was .8, effect was .3, and sample size was 5000. When the PC method, increases in sample size resulted in increased relative bias of standard errors estimates for all conditions, excluding entropy at .5, effect of 0, and sample size of 5000. When effect was 0 and sample size was 100, the PC method had the lowest average relative bias of standard errors estimates. Whereas, Vermunt’s method had the lowest average bias when effect was 0 and sample sizes were 500 and 5000. Vermunt’s method was also the lowest when effects were .3 and .5 and sample sizes were 100, 500, and 5000. As effect and sample size increases, bias decreases for Lanza’s and Vermunt’s methods and increases for the PC method.

Coverage. Sample size (G η^2 = .027) and effect (G η^2 = .009) had the largest main effects on coverage. The interactions with the largest effects were between effect, sample size, and method (G η^2 = .027), effect and method (G η^2 = .017), effect and sample size (G η^2 = .011). Interactions with the smallest effects were between entropy, sample size, and method, entropy and method, sample size and method, entropy, effect, sample size, and method, and entropy, effect, and method. There were mean increases in coverage with increases in sample size and increases in effect. The interaction between effect, sample size, and method show decreases in coverage for Lanza's and Vermunt's methods. Additionally, the interaction is the same for the PC method, excluding conditions where coverage was equal to 1 prior to increases. The PC method had the most acceptable levels of coverage in comparison to the other methods. Additionally, when examining the interaction between effect and method the PC method had an average coverage of 1 for each effect, except when effect was 0. Lanza's method decreased in levels of coverage with increases in sample size and effect. Vermunt's method and the PC method increased coverage with increases in sample size and effect.

Type I Error. The largest main effects on type I error rates were methods (G η2 = .054) and entropy (G η^2 = .007). The interaction between entropy and method also had an effect (G η^2 = .022). Levels of type I error rates ranged from .072 to .299 for Lanza's method, 0 to .028 for the PC method, and .038 to .065 for Vermunt's method. The PC method had the most acceptable levels of type I error rates between methods. Type I error rates were lower when entropy was valued at .8 compared to .5. Additionally, the interaction between entropy and method show that when entropy is increased Lanza's and Vermunt's methods do not have consistently have more acceptable type I error rates compared to the PC method at lower valued entropy.

Power. The main effects of sample size, method, effect, and entropy had the largest effect on power G η^2 = .286, .063, .047, and .043. The interactions between sample size and method, entropy and method, effect and sample size, entropy and sample size, and entropy, sample size, and method have smaller effects on power. Power averaged increases with higher sample size across methods. Power for Lanza's method ranged from .297 to 1, .028 to 1 for the PC method, and .128 to 1 for Vermunt's method. Power was most acceptable for Lanza's method compared to Vermunt's method and the PC method. Power increased with increases in effect. Power increased with increased values of entropy for all methods, except Lanza's method when entropy was .8, effect was .3, and sample size was 100.

DISCUSSION

The contribution of this research study is to provide a detailed comparison of the few methods of working with auxiliary variables in LCA and LPA, and a step-by-step of estimating the effects of class membership on distal outcomes with LPA and 3-step methods.

The Performance of 3-Step Methods with Auxiliary Predictor Variables

Since Asparouhov and Muthén's (2014) study is the only study that compares the three 3-step methods, it is useful to compare their results with the current study. Both studies agree that Vermunt's method has shown to be a better method in comparison to the PC method when the auxiliary variable is a latent class predictor. The power levels with Vermunt's method was higher than the power with the PC method. However, the current study indicates that the PC method has lower type 1-error rates under all studied conditions.

The current study expanded Asparouhov and Muthén's (2014) study by using different sample sizes, predictor variable and class relationships, and levels of entropy. Our results align with Clark and Muthén's (2009) study which showed that the PC method worked well only when entropy was greater than .8. . However, in this study, the PC method did not result in any acceptable levels of relative bias of coefficient estimates or standard error estimates under any manipulated conditions. Coverage was acceptable when effects were above zero, and type I error rates were acceptable at all levels of conditions.

Vermunt's method had satisfactory levels of relative bias of coefficient estimates for all conditions when sample size was above 100 except when effect and entropy were .5. Type I error rates were acceptable at all manipulations of conditions.

Vermunt's method provided acceptable results when entropy was .8, sample sizes were 500 and 5000, and effect was .3 and .5.

The Performance of 3-Step Methods with Distal Outcomes

Although the effects of sample size, B higher than zero, and entropy are not similar between the PC method and Vermunt's method when the auxiliary variable is a predictor of latent class, effects were not as definite in the distal outcome simulations. Asparouhov and Muthén's (2014) study reported that the PC method was outperformed by Vermunt's method, and thatLanza method is more accurate than Vermunt's method. This accuracy was due to Lanza's method not allowing for the distal outcome to significantly change the class membership for individual observations.

We found that the PC method outperforms both Lanza's and Vermunt's methods in terms of type I error when entropy is low. However, Vermunt's method had the best coverage under studied conditions. Lanza's method had lower relative bias of coefficient estimates, but Vermunt's and the PC method had lower relative bias of standard error estimates. Generally, our results match results of previous research and we agree that Lanza's method and Vermunt's method are better than the PC method. Since we included more sample sizes in this study than previous studies, we can conclude that when there is relationship between the auxiliary variable and the latent categorical variable, Lanza's method produces higher levels of power than Vermunt's method. However, Vermunt's method produced lower relative bias of standard error estimates.

Lanza's method had acceptable levels of relative bias of coefficient estimates at all manipulated conditions except when sample size was 100, effect was .3, and entropy was .5. Relative biases of standard error estimates were acceptable when entropy was

.8. Coverage was acceptable for all conditions except when entropy was .8 and sample size was 5000. Type I error rates were not acceptable under any manipulated conditions. Power was above 0.8 when sample size was above 500 and effect was .3. Also, power was above 0.8 when sample size was above 100 and effect was .5.

The PC method had no acceptable values of relative bias of coefficient estimates and standard error estimates. Additionally, relative biases of standard error estimates were acceptable when entropy was .8 and effect was zero. The estimate was also below .1 when entropy was .5, effect was .3, and sample size was 100. Coverage was acceptable for all conditions except when sample size was 5000 and effect was zero. Type I error rates were acceptable at all levels of manipulated conditions. Power was only consistently acceptable when sample sizes above 500.

Vermunt's method had acceptable values of relative bias of coefficient estimates across conditions, except when entropy was .5 and sample size was 100. Relative biases of standard error coefficients were only acceptable when entropy was .8. Coverage was above .95 across all conditions, except when sample size was 5000 and entropy was .8. Consistent acceptable values of type I error rates were at sample size of 500, and power was above 0.8 for only samples above 500.

Researchers using Lanza's method should expect entropy and sample size to decrease relative bias of coefficient estimates. Increases in entropy will decrease relative bias of standard error estimates. Coverage will increase with increases of entropy between .5 and .8. Additionally, sample size decreases type I error rates and increases power. For researchers using the PC method, expect relative bias of coefficient estimates and relative standard error bias to increase with sample sizes between 100 and 5000. Also, increases in effect size will increase coverage. Type I errors will decrease with increases of sample size, and higher entropy values will also increase type I error rates. For researchers using Vermunt's method, expect relative bias of coefficient estimates to decrease with sample size and entropy. Also, relative bias of standard error estimates decrease with entropy. Coverage will decrease with sample size, and power will increase with increases of sample size and values of entropy.

EXAMPLE APPLICATION OF LATENT PROFILE ANALYSIS

This example analysis demonstrates the use of the 3-step methods in latent profile analysis. The objective of the example analysis is to determine whether usage indicators for the Algebra Nation system can be to separate schools into classes, and whether these classes differ with respect to the passing rates in Florida's Algebra I End-of-Course assessment. Algebra Nation is an online learning environment to facilitate algebra learning. Participants were 1,185 middle and high schools in Florida during the 2014-2015 academic school year. All middle and high schools in the state were integrated with online access to Algebra Nation program and free workbooks. Schools had the option to use the program as little or as much as administrators, teachers, and students saw fit.

Method

Data was collected by the Algebra Nation system on number of workbooks ordered, teacher and student logins and videos, and test prep based on the Next Generation State Standards and Common Core Standards. Descriptive statistics show that Florida's schools offering Algebra I had a sum of 244,441 workbooks ordered, 1,726,348 logins, 1,801,521 video views, and 528,140 total test yourself prep.

Six Algebra Nation usage variables were used as indicators of the latent categorical variable, 1) average number of student logins, 2) average number of student videos viewed, 3) average number of teacher logins, 4) average number of teacher videos viewed, 5) average number of videos viewed by teachers and students, 6) average number of teacher and student logins.

The first step of the analysis was to identify the number of classes. We fit latent profile models with 2, 3 and 4 classes. These models were compared based on a set of fit indices: 1) AIC, 2) BIC, 3) the Vuong-Lo-Mendell Rubin adjusted likelihood ratio test, and 4) entropy.

The distal outcome for this analysis is the school-level Algebra I passing rate, which ranged in this sample from 3% to 99%. Estimation of the effect of latent classes on the distal outcome was conducted using the following approaches: PC, Vermunt's, and Lanza's methods. We used the 7.31 version of MPLUS to conduct each step of the analysis.

Results

Step 1: Determining classification accuracy of the models using model fit indices

The 2-class latent profile model was compared to the 3 and 4-class models. Model fit indices can be found in Table 3-16. The log-likelihood was replicated for each of the three models tested, which indicated that researchers had no evidence of reaching of local maxima (Blevins, Weathers, & Witte, 2014). As the number of classes extracted increased from 2 to 3 and 4, both AIC and BIC decreased as the number of tested classes increased. The 4-class model had the lowest AIC and BIC at 52953 and 53151. Entropy of models yielded values of .965 for the 2-class, .963 for the 3-class, and .971 for the 4-class. When comparing the 2-class model to a model with only one class and comparison of the 3-class model with the 2-class model, the LMR was below .05. However the LMR was .117 in favor of the 3-class model over the 4-class.

The 3-class model was chosen as the best fitting model on the basis that the LMR rejected the 4-class model and there was little differences between the fit indices of all three models. Researchers in applied fields should factor interpretable means and variances and number of classes along with fit indices when deciding a most appropriate model. For this study, we will only interpret the 3-class model.

Step 2: Identifying the classification solutions for 3- class model

Table 3-17 shows approximately, 46% of the schools belonged to the first class, 45% in the second class, and 9% in the third class. From Table 3-18, schools in the first class had a mean of 3.655 student logins, 3.184 student video views, 9.172 teacher logins, 7.754 teacher video views, 0.560 videos viewed by number of ordered workbooks, and 0.731 logins by number of ordered workbooks. While schools in the second class had a mean of 10.492 student logins, 13.043 student video views, 41.993 teacher logins, 73.415 teacher video views, 9.381 videos viewed by number of ordered workbooks, and 7.911 logins by number of ordered workbooks. Schools in the third and smallest class had a mean of 24.656 student logins, 38.673 student video views, 63.672 teacher logins, 215.482 teacher video views, 47.808 videos viewed by number of ordered workbooks, and 31.594 logins by number of ordered workbooks.

Step 3: Examining the probability of an outcome given latent profile membership

In this step we are interested in how school passing rates varied across classess of Algebra Nation usage. The PC, Vermunt's, and Lanza's methods were used to estimate mean differences across classes. The means for the PC method and Lanza's method were most similar, while Vermunt's method yielded higher mean passing rates for each class. Located in Table 3-19, the average passing rate using Lanza's method was 63.251 for the first class, 69.356 for the second class, and 80.992 for the third class; the PC method was slightly higher. Vermunt's method yielded class means of

63.374, 68.927, and 83.322 for passing rates. Additionally, a chi-square χ^2 test of independence was conducted to determine if each class was significantly different from each other. The mean differences between classes were all statisticaly significant at $\alpha = .05$ with all 3-step methods. Because classes one, two and three had increasing levels of usage of Algebra Nation, it can be concluded that higher levels of Algebra Nation usage corresponds to higher passing rates in the Algebra I End-of-Course exam..

CONCLUSION

This study made a comparison of three proposed methods for estimation of latent class analysis models with covariates. This early work sets the stage for the optimization and enhancement of these methods, and determination of when methods will yield optimal results.

Vermunt's method has shown to be the best available method for analyzing LCA models with covariates as the latent class predictor under a variety of conditions. Although this study was the second to make this conclusion, future studies should focus on the improvement of Vermunt's method to become more robust with conditions having little to no effect sizes.

Lanza's and Vermunt's methods have shown to be the best available approaches for estimating the effect of latent class membership on a distal outcome. In Asparouhov & Muthén (2014) and Lanza et al. (2013), Lanza's method was determined the best method based on coverage, bias, and mean squared error. Between the Vermunt's and Lanza's methods, the preferred method which yielded the most acceptable parameters estimates in the conditions evaluated in this study was Vermunt's method. As suggested in Lanza et al. (2013) our study has contributed by varying the effects of the class on the distal outcome. Future research should contribute by improving these methods' parameter estimates when effect and sample size is small, testing these approaches in latent profile analysis, growth mixture modeling, factor mixture modeling, and mixture regression modeling (Lanza et al., 2013).

Although there are individual cases where the PC method resulted in acceptable levels for each parameter estimate, no tested set of manipulated conditions produced all acceptable estimates. At this point in the research of this method, there is not a proven set of conditions where the PC method will work with predictor auxiliary variables in LCA. However, the PC method does yield acceptable results across all studied parameters when there is no effect and entropy is .8 for distal outcomes. Unfortunately, in most applied studies the researchers will be testing for effect; in such cases, the PC method would still not be useful.

Vermunt's method showed to be a reliable method; however, the approach came with drawbacks. The method yields acceptable parameter estimates when there is an effect of predictor auxiliary variables. Alternatively, researchers using this method will lose coverage and increase relative bias of standard error estimates in cases where there is no effect. In terms of the distal outcome, Vermunt's method yields acceptable results when entropy is .8. Researchers using this method should note that when sample size is 5000, coverage would lessen slightly.

Although Lanza's method showed to have the highest power amongst the studied methods, levels of entropy affect this approach and outputs higher than desired type I error rates. Because of the type I error results, researchers using Lanza's method should expect from previous literature that the distal outcome has an effect of greater than or equal to .3.

In the applied study, the Vermunt's method yielded slightly higher values of means across classes and *p* values when comparing classes in the χ^2 test. Applied researchers implementing LPA with distal outcomes in their analyses should be aware of these differences when values are at cut-offs critical to theory. More studies are needed to compare LPA models with covariates in applied and simulation studies.

This study compared methods under different conditions for using auxiliary variables in LCA models. However, no simulations in this study had missing data, so future studies should compare the three methods with missing data. The three methods studied were not compared to the original, one-step method. Future studies should also include a simulation of LPA and apply entropy greater than .8 to determine the effect of optimal class separation. Based on previous research, increasing entropy should improve the parameter estimates of the PC method.

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Table 3-1. Effect of manipulated conditions on relative bias with predictor auxiliary variables

Effect	**G η^2**
Method	0.148
Entropy	0.072
Entropy X Method	0.044
Effect	0.005
Sample Size	0.003
Effect X Sample Size	0.003
Sample Size X Method	0.001
Entropy X Sample Size X Method	0.001
Entropy X Effect	0.000
Entropy X Sample Size	0.000
Effect X Sample Size X Method	0.000
Entropy X Effect X Method	0.000
Entropy X Effect X Sample Size	0.000
Effect X Method	0.000
Entropy X Effect X Sample Size X Method	0.000

Table 3-2. Effect of manipulated conditions on relative bias of standard error estimates with predictor auxiliary variables

Effect	**G η^2**
Effect	0.646
Method	0.578
Entropy	0.379
Entropy X Method	0.351
Effect X Method	0.053
Entropy X Effect X Method	0.023
Sample Size X Method	0.015
Entropy X Sample Size X Method	0.015
Entropy X Effect	0.007
Sample X Size	0.005
Entropy X Sample Size	0.004
Effect X Sample Size	0.003
Entropy X Effect Sample Size	0.001
Effect X Sample Size X Method	0.001
Entropy X Effect X Sample Size X Method	0.001

Table 3-3. Effect of manipulated conditions on coverage with predictor auxiliary variables

Effect	**G η^2**
Effect	0.103
Effect X Method	0.001
Method	0.001
Effect X Sample Size	0.000
Entropy X Effect X Sample Size	0.000
Entropy	0.000
Sample Size	0.000
Entropy X Method	0.000
Entropy X Effect X Method	0.000
Entropy X Effect X Sample Size X Method	0.000
Entropy X Effect	0.000
Sample Size X Method	0.000
Entropy X Sample Size	0.000
Effect X Sample Size X Method	0.000
Entropy X Sample Size X Method	0.000

Table 3-4. Effect of manipulated conditions on type I error with predictor auxiliary variables

Effect	**G η^2**
Method	0.011
Entropy	0.002
Entropy X Method	0.001
Entropy X Sample Size X Method	0.000
Sample Size	0.000
Sample Size X Method	0.000
Entropy X Sample Size	0.000

Table 3-5. Effect of manipulated conditions on power with predictor auxiliary variables

Effect	**G η^2**
Sample Size	0.389
Entropy	0.117
Entropy X Sample Size	0.058
Effect	0.042
Effect X Sample Size	0.020
Method	0.020
Sample Size X Method	0.009
Entropy X Method	0.007
Entropy X Sample Size X Method	0.003
Entropy X Effect X Method	0.000
Entropy X Effect X Sample Size X Method	0.000
Entropy X Effect X Sample Size	0.000
Effect X Method	0.000
Effect X Sample Size X Method	0.000
Entropy X Effect	0.000

Table 3-6. Monte Carlo simulation results with pseudo class method to estimate effect of a latent class predictor

Method	Entropy	Effect	Sample Size	Relative Bias of Coefficient Estimate	Relative Bias of Standard Error Estimate	Coverage	Type I Error	Power
PC	0.500	0.000	100.000	-	1.449	0.778	0.002	-
PC	0.500	0.000	500.000	-	1.948	0.759	0.001	-
PC	0.500	0.000	5000.000	-	2.033	0.768	0.000	-
PC	0.500	0.300	100.000	-0.355	0.965	0.989	-	0.012
PC	0.500	0.300	500.000	-0.556	0.823	1.000	-	0.069
PC	0.500	0.300	5000.000	-0.568	0.920	1.000	-	0.999
PC	0.500	0.500	100.000	-0.482	0.640	0.994	-	0.043
PC	0.500	0.500	500.000	-0.570	0.807	1.000	-	0.398
PC	0.500	0.500	5000.000	-0.578	0.905	1.000	-	1.000
PC	0.800	0.000	100.000	-	0.839	0.782	0.024	-
PC	0.800	0.000	500.000	-	1.020	0.761	0.018	-
PC	0.800	0.000	5000.000	-	0.949	0.746	0.023	-
PC	0.800	0.300	100.000	-0.030	0.236	0.970	-	0.180
PC	0.800	0.300	500.000	-0.130	0.162	0.988	-	0.780
PC	0.800	0.300	5000.000	-0.162	0.175	1.000	-	1.000
PC	0.800	0.500	100.000	-0.125	0.201	0.984	-	0.435
PC	0.800	0.500	500.000	-0.165	0.164	0.994	-	0.994
PC	0.800	0.500	5000.000	-0.165	0.193	1.000	-	1.000

Table 3-7. Monte Carlo simulation results with Vermunt's method to estimate effect of a latent class predictor

Method	Entropy	Effect	Sample Size	Relative Bias of Coefficient Estimate	Relative Bias of Standard Error Estimate	Coverage	Type I Error	Power
Vermunt	0.500	0.000	100.000	-	0.618	0.793	0.031	-
Vermunt	0.500	0.000	500.000	-	0.611	0.774	0.044	-
Vermunt	0.500	0.000	5000.000	-	0.611	0.779	0.055	-
Vermunt	0.500	0.300	100.000	0.104	0.284	0.955	-	0.074
Vermunt	0.500	0.300	500.000	-0.051	0.039	0.969	-	0.414
Vermunt	0.500	0.300	5000.000	-0.002	0.022	0.970	-	1.000
Vermunt	0.500	0.500	100.000	-0.112	0.071	0.965	-	0.163
Vermunt	0.500	0.500	500.000	-0.064	-0.014	0.975	-	0.817
Vermunt	0.500	0.500	5000.000	-0.001	0.016	0.976	-	1.000
Vermunt	0.800	0.000	100.000	-	0.602	0.782	0.053	-
Vermunt	0.800	0.000	500.000	-	0.710	0.778	0.044	-
Vermunt	0.800	0.000	5000.000	-	0.677	0.760	0.050	-
Vermunt	0.800	0.300	100.000	0.126	0.075	0.953	-	0.258
Vermunt	0.800	0.300	500.000	0.035	-0.011	0.966	-	0.844
Vermunt	0.800	0.300	5000.000	-0.004	0.009	0.978	-	1.000
Vermunt	0.800	0.500	100.000	0.026	0.018	0.965	-	0.535
Vermunt	0.800	0.500	500.000	0.001	-0.024	0.969	-	0.997
Vermunt	0.800	0.500	5000.000	0.002	0.010	0.976	-	1.000

Table 3-8. Effect of manipulated conditions on relative bias with distal outcome variables

Effect	**G η^2**
Method	0.279
Sample Size X Method	0.004
Entropy X Method	0.002
Entropy X Sample Size X Method	0.001
Entropy	0.001
Sample Size	0.000
Effect	0.000
Effect X Sample Size X Method	0.000
Entropy X Effect	0.000
Effect X Method	0.000
Entropy X Effect X Method	0.000
Entropy X Sample Size	0.000
Effect X Sample Size	0.000
Entropy X Effect X Sample Size X Method	0.000
Entropy X Effect X Sample X Size	0.000

Table 3-9. Effect of manipulated conditions on relative bias of standard error estimates with distal outcome variables

Effect	**G η^2**
Effect X Method	0.754
Entropy X Method	0.605
Effect	0.600
Sample Size X Method	0.308
Effect X Sample Size X Method	0.178
Entropy	0.149
Method	0.145
Sample Size	0.101
Effect X Sample Size	0.094
Entropy X Sample Size X Method	0.035
Entropy X Effect X Sample Size X Method	0.024
Entropy X Effect X Method	0.018
Entropy X Effect	0.009
Entropy X Effect X Sample Size	0.006
Entropy X Sample Size	0.001

Table 3-10. Effect of manipulated conditions on coverage with distal outcome variables

Effect	**G η^2**
Sample Size	0.027
Effect X Sample Size X Method	0.020
Effect X Method	0.017
Effect X Sample Size	0.011
Effect	0.009
Entropy X Sample Size X Method	0.005
Entropy X Method	0.004
Sample Size X Method	0.002
Entropy X Effect X Sample Size X Method	0.002
Entropy X Effect X Method	0.002
Method	0.001
Entropy X Sample Size	0.001
Entropy X Effect	0.000
Entropy	0.000
Entropy X Effect X Sample Size	0.000

Table 3-11. Effect of manipulated conditions on type I error with distal outcome variables

Effect	**G η^2**
Method	0.054
Entropy X Method	0.022
Entropy	0.007
Sample Size X Method	0.001
Sample Size	0.000
Entropy X Sample Size X Method	0.000
Entropy X Sample Size	0.000

Table 3-12. Effect of manipulated conditions on power with distal outcome variables

Effect	**G η^2**
Sample Size	0.286
Method	0.063
Effect	0.047
Entropy	0.043
Sample Size X Method	0.032
Entropy X Method	0.030
Effect X Sample Size	0.024
Entropy X Sample Size	0.020
Entropy X Sample Size X Method	0.014
Entropy X Effect X Method	0.000
Effect X Method	0.000
Entropy X Effect X Sample Size X Method	0.000
Entropy X Effect X Sample Size	0.000
Effect X Sample Size X Method	0.000
Entropy X Effect	0.000

Table 3-13. Monte Carlo simulation study with a continuous distal outcome auxiliary variable for conditions with zero effect

Method	Entropy	Effect	Sample Size	Relative Bias of Standard Error Estimates	Coverage	Type I Error
Lanza	0.5	0.000	100.000	-0.522	0.989	0.299
Lanza	0.5	0.000	500.000	-0.341	0.996	0.186
Lanza	0.5	0.000	5000.000	-0.349	0.984	0.200
PC	0.5	0.000	100.000	0.269	0.982	0.005
PC	0.5	0.000	500.000	0.578	0.955	0.001
PC	0.5	0.000	5000.000	0.539	0.690	0.000
Vermunt	0.5	0.000	100.000	-0.268	0.997	0.065
Vermunt	0.5	0.000	500.000	-0.264	0.996	0.038
Vermunt	0.5	0.000	5000.000	-0.294	0.986	0.059
Lanza	0.8	0.000	100.000	-0.119	0.996	0.093
Lanza	0.8	0.000	500.000	-0.071	0.993	0.077
Lanza	0.8	0.000	5000.000	-0.056	0.911	0.072
PC	0.8	0.000	100.000	0.068	0.995	0.028
PC	0.8	0.000	500.000	0.113	0.983	0.022
PC	0.8	0.000	5000.000	0.135	0.830	0.019
Vermunt	0.8	0.000	100.000	-0.092	0.995	0.059
Vermunt	0.8	0.000	500.000	-0.055	0.995	0.045
Vermunt	0.8	0.000	5000.000	-0.044	0.913	0.051

Table 3-14. Monte Carlo simulation study with a continuous distal outcome auxiliary variable with effect of .3

Method	Entropy	Effect	Sample Size	Relative Bias of Coefficient Estimate	Relative Bias of Standard Error Estimates	Coverage	Power
Lanza	0.5	0.300	100.000	0.066	-0.507	0.987	0.424
Lanza	0.5	0.300	500.000	0.030	-0.371	0.995	0.812
Lanza	0.5	0.300	5000.000	0.003	-0.330	0.977	1.000
PC	0.5	0.300	100.000	-0.705	0.068	0.998	0.028
PC	0.5	0.300	500.000	-0.857	-0.336	1.000	0.081
PC	0.5	0.300	5000.000	-0.918	-0.772	1.000	1.000
Vermunt	0.5	0.300	100.000	-0.239	-0.266	0.998	0.128
Vermunt	0.5	0.300	500.000	-0.047	-0.313	0.994	0.479
Vermunt	0.5	0.300	5000.000	0.002	-0.279	0.984	1.000
Lanza	0.8	0.300	100.000	-0.053	-0.081	0.995	0.297
Lanza	0.8	0.300	500.000	0.021	-0.028	0.984	0.918
Lanza	0.8	0.300	5000.000	-0.005	-0.099	0.939	1.000
PC	0.8	0.300	100.000	-0.720	-0.299	1.000	0.165
PC	0.8	0.300	500.000	-0.779	-0.653	1.000	0.811
PC	0.8	0.300	5000.000	-0.824	-0.884	1.000	1.000
Vermunt	0.8	0.300	100.000	-0.086	-0.067	0.997	0.221
Vermunt	0.8	0.300	500.000	0.019	-0.015	0.985	0.881
Vermunt	0.8	0.300	5000.000	-0.005	-0.086	0.946	1.000

Table 3-15. Monte Carlo simulation study with a continuous distal outcome auxiliary variable with effect of .5

Method	Entropy	Effect	Sample Size	Relative Bias of Coefficient Estimate	Relative Bias of Standard Error Estimates	Coverage	Power
Lanza	0.5	0.500	100.000	0.090	-0.470	0.988	0.664
Lanza	0.5	0.500	500.000	0.003	-0.347	0.992	0.989
Lanza	0.5	0.500	5000.000	-0.001	-0.321	0.971	1.000
PC	0.5	0.500	100.000	-0.710	-0.189	0.998	0.070
PC	0.5	0.500	500.000	-0.856	-0.559	1.000	0.462
PC	0.5	0.500	5000.000	-0.911	-0.860	1.000	1.000
Vermunt	0.5	0.500	100.000	-0.183	-0.281	0.997	0.297
Vermunt	0.5	0.500	500.000	-0.079	-0.310	0.995	0.862
Vermunt	0.5	0.500	5000.000	0.002	-0.285	0.979	1.000
Lanza	0.8	0.500	100.000	-0.007	-0.095	0.994	0.675
Lanza	0.8	0.500	500.000	-0.006	-0.084	0.995	0.999
Lanza	0.8	0.500	5000.000	0.001	-0.053	0.919	1.000
PC	0.8	0.500	100.000	-0.734	-0.535	1.000	0.477
PC	0.8	0.500	500.000	-0.749	-0.777	1.000	0.996
PC	0.8	0.500	5000.000	-0.804	-0.930	1.000	1.000
Vermunt	0.8	0.500	100.000	-0.035	-0.079	0.995	0.554
Vermunt	0.8	0.500	500.000	-0.007	-0.067	0.991	0.999
Vermunt	0.8	0.500	5000.000	0.000	-0.042	0.928	1.000

Table 3-16. Fit Indices for Competing Latent Profile Models

Model	Log-Likelihood	AIC	BIC	Entropy	LMR-A *p-value*
2 class	-27110.020	54270.039	54396.977	0.965	<.001
3 class	-26677.187	53418.373	53580.853	0.963	.008
4 class	-26437.697	52953.395	53151.417	0.971	0.1166

Table 3-17. Class Counts and Proportions for 3-class Model

Latent Classes	Counts	Proportions
1	545.091	.460
2	536.005	.452
3	103.903	.088

Table 3-18. Latent Profile Model Results

Latent Class	Indicator (Mean)	Estimate	Variance	p value
1 class	U1	3.655	5.595	<.05
	U2	3.184	9.747	<.05
	U3	9.172	121.735	<.05
	U4	7.754	142.683	<.05
	U5	.560	0.766	<.05
	U6	.731	1.242	<.05
2 class	U1	10.492	40.402	<.05
	U2	13.043	117.971	<.05
	U3	41.993	2321.228	<.05
	U4	73.415	40374.367	<.05
	U5	9.381	153.970	<.05
	U6	7.911	150.246	<.05
3 class	U1	24.656	40.402	<.05
	U2	38.673	117.971	<.05
	U3	63.672	2321.228	<.05
	U4	215.482	40374.367	<.05
	U5	47.808	153.970	<.05
	U6	31.594	150.246	<.05

Table 3-19. Equality Test of Means using 3-Step Methods

Method	Latent Class	Mean	Overall Test	χ^2	p value
PC	1 class	63.359	Class 1 vs 2	15.629	<.001
PC	2 class	69.466	Class 1 vs 3	48.273	<.001
PC	3 class	80.249	Class 2 vs 3	19.944	<.001
Vermunt	1 class	63.374	Class 1 vs 2	30.251	.001
Vermunt	2 class	68.927	Class 1 vs 3	10.607	.001
Vermunt	3 class	83.322	Class 2 vs 3	5.012	.025
Lanza	1 class	63.251	Class 1 vs 2	71.065	<.001
Lanza	2 class	69.356	Class 1 vs 3	15.802	<.001
Lanza	3 class	80.992	Class 2 vs 3	70.313	<.001